Convertible Bond
A convertible bond can be thought of as a normal corporate bond with embedded options, which enable the holder to exchange the bond for the issuer's stock. Having properties of both stocks and bonds, convertibles can be an attractive choice for investors and have tended to have lower risk.

Issuers have several reasons to use convertible financing. By issuing convertibles they can lower their cost of debt funding compared to straight debt alone. Lower-credit companies who may not be able to access the straight debt market can often still issue convertible debt. Companies who anticipate equity appreciation can use convertibles to defer equity financing to a time when growth has been achieved.

Investors find several features of convertibles appealing. They offer greater satiability of income than common stock. They provide a yield that is often higher than the dividend yield of common stock. Finally, because they are often theoretically underpriced, they may provide a cheap source of common stock volatility.
Convertible

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Convertible Bond Introduction

- A convertible bond has an embedded call option that gives bondholders the right to convert their bonds into equity at a given time for a predetermined number of shares in the issuing company.
- Convertible bonds typically have lower yields than the yields on similar bonds without the convertible option.
- Most convertible bonds are subordinated debt of the issuer. In the event of bankruptcy, the claims of other bondholders take priority over convertible bondholders except the preferred and common stock owners.
Convertible bonds

- By issuing convertibles the companies can lower their cost of debt funding compared to straight debt alone.
- Lower-credit companies who may not be able to access the straight debt market can often still issue convertible debt.
- Companies who anticipate equity appreciation can use convertibles to defer equity financing to a time when growth has been achieved.
- Convertible bonds offer greater satiability of income than common stock.
- They provide a yield that is often higher than the dividend yield of common stock.
- Given the optionality, convertibles have tended to have lower risk.
Convertible bonds are hybrid securities that have both debt and equity features.

The valuation of convertible or reverse convertible bonds can be quite complex because of its dual nature as a normal bond and as an equity call/put option.

There is no closed-form solution for convertibles.

Convertible prices can only be solved by numerical methods, such as, Monte Carlo simulation, tree/lattice approaches, or partial differential equation (PDE) solutions.

The valuation of a convertible bond normally has a backward nature since there is no way of knowing whether the convertible should be converted without knowledge of the future value.
Pricing (Con’t)

- Three sources of randomness exist in a convertible bond: the stock price, the interest rate, and the credit spread.
- Interest rate is assumed to be constant as the effect of a stochastic interest rate on convertible bond prices is so small that it can be neglected.
- Accurately modeling the equity process appears crucial.
- Since convertible bonds are issued mainly by start-up or small companies, credit risk plays an important role in the valuation.
- FinPricing uses PDE to price convertible and reverse convertible bonds, and use Monte Carlo simulation to value convertibles with exotic path-dependent trigger provisions.
The value of the convertible at each node is divided into two components: a component of bond and a component of stock.

The PDE of the equity component $G$ is given by

$$\frac{\partial G}{\partial t} + 0.5\sigma^2 S^2 \frac{\partial^2 G}{\partial S^2} + (r - q + h(1 - \varphi_s))S \frac{\partial G}{\partial S} - (r + h(1 - \varphi_s))G = 0$$

where

- $S$ the stock price
- $r$ the interest rate
- $q$ the dividend
- $h$ the hazard rate
- $\varphi_s$ the equity recovery rate
Pricing (Con’t)

- The PDE of the bond component $B$ is
  \[
  \frac{\partial B}{\partial t} + 0.5 \sigma^2 S^2 \frac{\partial^2 B}{\partial S^2} + (r - q + h(1 - \varphi_s)) S \frac{\partial B}{\partial S} - (r + h(1 - \varphi_b))B = 0
  \]
  where $\varphi_b$ is the bond recovery rate

- The final conditions at maturity $T$ can be generalized as
  \[
  G_T = \begin{cases} 
  \eta S_T & \text{if } \eta S_T > \min[P_c, \max(P_p, N + C)] \\
  0 & \text{otherwise}
  \end{cases}
  \]

  \[
  B_T = \begin{cases} 
  \min[P_c, \max(P_p, N + C)] & \text{if } \eta S_T \leq \min[P_c, \max(P_p, N + C)] \\
  0 & \text{otherwise}
  \end{cases}
  \]

  where $N$ denotes the bond principal, $C$ denotes the coupon, $P_c$ denotes the call price, $P_p$ denotes the put price and $\varphi$ denotes the conversion ratio.
The valuation can be done via backward induction. The procedure is as follows.

For \( i = \text{penultimateTime to current Time} \)

- determine accrual interest and call/put prices
- determine boundary nodes
- use the PSOR (Projected Successive over Relaxation) method to obtain the
  - continuation value of the bond component
  - and the continuation value of the equity component, applying the constraints.

EndFor

- The value at node[0][y] is the convertible bond price where the equity price at node[0][y] is equal to the current market stock price.
### Convertible Example

<table>
<thead>
<tr>
<th>Underlying Equity</th>
<th>EFN.TO</th>
<th>Conversion Ratio</th>
<th>52.35</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Convertible Bond ISIN</strong></td>
<td>CA286181AB88</td>
<td>Conversion Start Date</td>
<td>1/1/2016</td>
</tr>
<tr>
<td><strong>Currency</strong></td>
<td>CAD</td>
<td>Conversion End Date</td>
<td>6/30/2020</td>
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<tr>
<td><strong>Face Value</strong></td>
<td>100</td>
<td>Coupon</td>
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<td><strong>Percent Redemption</strong></td>
<td>1</td>
<td>First Coupon Date</td>
<td>12/31/2015</td>
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<td><strong>DayCount</strong></td>
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<td>First Settle Date</td>
<td>5/29/2015</td>
</tr>
<tr>
<td><strong>Call Start Date</strong></td>
<td>6/30/2018</td>
<td>Coupon Frequency</td>
<td>SEMIANNUAL</td>
</tr>
<tr>
<td><strong>Call End Date</strong></td>
<td>6/29/2020</td>
<td>Roll Type</td>
<td>Following</td>
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<tr>
<td><strong>Call Price</strong></td>
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<td>Issue Price</td>
<td>100</td>
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<tr>
<td><strong>Call Notice Period</strong></td>
<td>30</td>
<td>Market Quote Type</td>
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</tr>
<tr>
<td><strong>Call Trigger</strong></td>
<td>125</td>
<td>Maturity</td>
<td>6/30/2020</td>
</tr>
<tr>
<td><strong>Trigger days</strong></td>
<td>20</td>
<td>Par Amount</td>
<td>1000</td>
</tr>
</tbody>
</table>
Reference:

https://finpricing.com/lib/EqConvertible.html